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EXISTENCE OF SOLUTIONS OF KRAIKO'S PROBLEM

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Three initial-boundary-value problems for the equations of gas dynamics are formulated. Successive solution of these problems yields a solution of Kraiko's problem of the isentropic transition of an ideal gas from a homogeneous state of rest to another state of rest with higher or lower density. Solutions are constructed for plane, cylindrical, and spherical layers of an ideal gas. The existence of locally analytic solutions is proved.

Introduction. For the plane-symmetric case, Mises [1] considered an example of a compound flow of a gas in which transition from a homogeneous state of rest (state 1) to another state of rest (state 2) is performed by means of two centered waves and a constant flow. This flow is produced by the motion of two impermeable pistons. It has been proven that solutions of this type do not exist if one of the pistons remains at rest (see [1]).

Rylov [2] studied the problem of the optimal motion of an impermeable piston that does maximum work under specified constraints on the displacement and the displacement time. For the plane-symmetric case, Kraiko [3] obtained numerical results and drew an analogy between the optimal motion of the piston and the well-known motion in a two-dimensional supersonic nozzle of maximum thrust.

Kraiko [4–7] constructed compound plane, cylindrically, and spherically symmetric, unsteady gas flows that describe the isentropic transition of an ideal gas from state 1 to state 2 with higher or lower density ρ (compression wave or rarefaction wave, respectively) and used them, in particular, to describe unlimited cumulation of a gas. These flows differ from the configuration considered in [1]. The compound flow configuration in [5, 7] for the shock-free compression of a gas to finite density also differs from the configuration considered in [8]. For the compression of a gas to finite density, Sidorov [8] studied a configuration in which the characteristics of one family intersect at a point that lies on the stationary boundary of the compressed layer and not on the piston. In state 2, the gas density can be constant but the gas-flow velocity is then nonzero and the flow configuration is similar to the one described in [1]. Kraiko [5, 7] considered flows that arise when a compressing piston comes to the point from which a centered compression wave propagates. In this case, in state 2, the density is constant and the gas velocity is zero. In [4–7], approximate formulas were used to resolve singularities in the solution, and construction of the flow "as a whole" was reduced to a numerical solution of the equations of gas dynamics by the method of characteristics. We note that solutions of problems of the strong shock-free compression of an ideal gas are unstable with respect to external actions on the gas [9].

The aim of the present paper is to formulate three initial-boundary-value problems and prove the existence of locally analytic solutions of these problems. Successive solution of these problems yields solutions of Kraiko's problem for plane ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) layers of an ideal gas. Thus, for $\nu = 0$, 1, and 2, we prove that a gas mass can be transferred from state 1 to state 2 by a shock-free method using an impermeable piston. Also, we discuss the arbitrariness encountered in solving Kraiko's problem and the possibility of non-one-dimensional flows in this problem.

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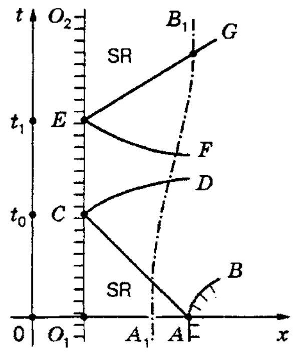

Fig. 1

Initial-Boundary-Value Problems. The equations of gas dynamics are invariant with respect to a shift in time t and inversions of the gas velocity and time [10]. Therefore, without loss of generality, we consider only the case of a rarefaction wave in Kraiko's problem.

To solve Kraiko's problem, it is necessary to study isentropic flows of an ideal gas ($S = S_0$, where S is the entropy and $S_0 = \text{const} > 0$). For simplicity, we consider the polytropic equation of state $p = A^2(S)\rho^\gamma/\gamma$, where p is the pressure and $\gamma = \text{const} > 1$. Without loss of generality, we assume that $A^2(S_0) = 1$. However, all the theorems proved below can easily be extended to the case of a normal gas with an arbitrary equation of state $p = p(S, \rho)$ if the function $p(S_0, \rho)$ is analytic in the neighborhood of the examined point $\rho = \rho_0$ and $p(S_0, \rho_0) > 0$, where $\rho_0 = \text{const} > 0$.

To describe one-dimensional isentropic flows of an ideal polytropic gas in Kraiko's problem, we consider the following system of equations for the gas velocity u and the speed of sound c:

$$c_t + uc_x + (\gamma - 1)c(u_x + \nu u/x)/2 = 0, \qquad u_t + 2cc_x/(\gamma - 1) + uu_x = 0 \tag{1.1}$$

and the equation for the velocity potential $\Phi(t, x)$ [10]. To describe flow singularities such as centered waves, we convert the function Φ to a new unknown function $\Psi(t, u)$ using the Legendre transform $\Phi = -\Psi + ux + (\gamma - 1)t$. The Jacobian of this transform is given by $J = -\Psi_{uu}$. The equation for the function Ψ has the form

$$\Psi_{tt}\Psi_{uu} - (\Psi_{tu} - u)^2 + c^2 + \nu uc^2\Psi_{uu}/\Psi_u = 0. \tag{1.2}$$

Here $c^2 = (\gamma - 1)(\Psi_t - u^2/2)$, conversion to the space of physical variables is performed using the formula $x = \Psi_u$, where $x = x_1$ for $\nu = 0$ and $x = \Big(\sum\limits_{i=1}^{\nu+1} x_i^2\Big)^{1/2}$ for $\nu = 1$ and 2, and t is time.

For system (1.1) and Eq. (1.2), we formulate three initial-boundary-value problems. Let us consider a plane, cylindrical, or spherical layer of a homogeneous gas ($\rho = \rho_0 = 1$) which at $t = 0$ is at rest between two impermeable walls located at the points $O_1(x = x_0)$ and $A(x = x_*)$. Without loss of generality, we set $x_* = 1$. For definiteness, we assume that the point O_1 is to the left of the point A ($x_0 < x_*$) but for $\nu = 1$ and $\nu = 2$, it is strictly to the right of the axis or center of symmetry, respectively ($x_0 > 0$).

Problem 1 (of a piston moving out). Let the wall move out as an impermeable piston. Three configurations are possible, depending on how the piston moves.

We assume that for $t \geqslant 0$, the piston moves smoothly starting at the point A (Fig. 1). Its trajectory (AB) is specified by the equation $x = x_p(t)$ [$x_p(0) = x_*$, $x_p'(0) = 0$, and $x_p''(0) > 0$]. Then, for $t \geqslant 0$, two flows are matched continuously via the sonic characteristic AC ($x = x_* - t$) in the region between the motionless wall O_1O_2 and the piston AB. The region O_1AC corresponds to a state of rest of the gas (the region SR in Fig. 1), and the region BAC corresponds to a flow, which is uniquely determined [9] from the characteristic Cauchy problem for Eq. (1.2):

$$\Psi(t, u)\Big|_{u=0} = (x_* - t)/(\gamma - 1), \qquad \Psi_u(t, u)\Big|_{u=0} = x_* - t; \tag{1.3}$$

$$\Psi_{uu}(t,u)\Big|_{t=\eta(u)} = \eta'(u)[u - \Psi_{tu}(t,u)]\Big|_{t=\eta(u)}. \tag{1.4}$$

Here $t = \eta(u)$ is the inverse of the function $u = x_p'(t)$. Equations (1.3) represent the condition of continuous matching of the solution of problem (1.2)–(1.4) and the solution corresponding to the homogeneous state of rest via the characteristic AC. Equation (1.4) represents the condition of no normal flow through the piston AB for the function $\Psi(t,u)$. Problem (1.2)–(1.4) is Problem 1.

Theorem 1. *If the function $t = \eta(u)$ is analytic in a certain neighborhood of the point $u = 0$, Problem 1 has a unique analytic solution which is defined in a certain neighborhood of any point ($t = t_*$ and $u = 0$) such that $x_* - t_* > 0$. In this case, $J \neq 0$.*

The proof of the Theorem 1 includes two steps. At the first step, it is shown that Problem 1 is a characteristic Cauchy problem of standard form [9], and this ensures the existence of an analytic solution in a certain neighborhood of the point ($t = 0$, $u = 0$). It is proved that there exists a neighborhood of the point ($t = 0$, $u = 0$) in which $J \neq 0$. At the second step of the proof, the coefficients of the series that specify the solution of Problem 1 are studied in detail and it is established [9] that the solution exists in a certain neighborhood of the semiaxis $u = 0$ for $t < x_*$ and $J \neq 0$ in this neighborhood.

Thus, it has been proven that in the space of independent variables (t,x), the problem of a piston moving out smoothly from a motionless homogeneous gas has a solution in the class of piecewise-analytic functions, and this solution is unique and defined in a certain neighborhood of the sonic characteristic AC.

Let the piston move out suddenly with a velocity not lower than the velocity of a gas flow into vacuum: $x_p'(0) \geqslant 2/(\gamma - 1)$ (the problem of a gas flow into vacuum [10]). In this case for small $t \geqslant 0$, a homogeneous state of rest is attached via the sonic characteristic AC to a simple centered Riemann wave [10] for $\nu = 0$ and to a flow similar to a centered wave [9] for $\nu = 1$ and 2. This flow is a solution of Problem 1 in which the constant $t = 0$ is used instead of the function $t = \eta(u)$. In this case, condition (1.4) takes the form $\Psi_{uu}(0,u) = 0$ and, in the space (t,x), it corresponds to instantaneous removal of the wall from the point A. Thus, Theorem 1 is valid for the problem of a gas flow into vacuum, and $J \neq 0$ in the entire indicated set of points, except at the origin: $J\Big|_{t=u=0} = 0$.

A third possible configuration in Problem 1 is produced by fast (but inadequate to give rise to vacuum) motion of an impermeable piston from the point A: $0 < x_p'(0) < 2/(\gamma - 1)$. For general spatial flows, it has been proven [11, 12] that if the law of motion of a piston is analytic, a solution of this problem exists in a certain neighborhood of the point A, is unique in the class of piecewise-analytic functions, and consists of three flows separated by sonic characteristics. One of these flows, located in the region O_1AC, corresponds to a homogeneous state of rest. The second is a generalization of a centered Riemann wave and has a specific singularity at the initial time. The third flow is, in fact, a solution of the problem of the smooth motion of a piston from the specified flow and has no singularities in a certain neighborhood of the point A. Therefore, in the third case, an analytic solution is also defined in a certain neighborhood of the characteristic AC [11, 12].

Thus, for any analytic law of motion of an impermeable piston from the point A, the solution corresponding to a rarefaction wave is uniquely determined, is matched continuously via the sonic characteristic AC to the solution corresponding to the initial homogeneous state of rest, and is given by analytic functions in a certain neighborhood of the point $C(t = t_0,\ x = x_0)$, where $t_0 = x_* - x_0$.

Problem 2 (of the reflection of a rarefaction wave from a rigid wall). In the solution of Problem 1, the gas velocity is strictly lower than zero at $x = x_0$ and $t > t_0$. Therefore, this solution does not satisfy the condition of no normal flow through the wall O_1O_2. Hence, in Problem 1, the interaction of the flow with the wall O_1O_2 results in a new flow which is separated from the former by the sonic characteristic CD from the family of characteristics C^+ (Fig. 1). The function $x = \varphi(t)$ that defines the characteristic CD and the gas parameters on this characteristic

$$c\Big|_{x=\varphi(t)} = c_0(t), \qquad u\Big|_{x=\varphi(t)} = u_0(t) \tag{1.5}$$

are uniquely determined by solving Problem 1, and they are analytic functions in a certain neighborhood of the point $t = t_0$. In this case, $\dot{\varphi}(t) = u_0(t) + c_0(t)$ and $\varphi(t_0) = x_0$. The new flow must satisfy the conditions

(1.5) of continuous matching to the solution of Problem 1 via the characteristic CD. At the wall O_1O_2, this flow must satisfy the condition of no normal flow:

$$u\Big|_{x=x_0} = 0. \tag{1.6}$$

Problem (1.1), (1.5), (1.6) is Problem 2.

Theorem 2. *Problem 2 has a unique analytic solution in a certain neighborhood of the point* ($t = t_0$, $x = x_0$).

To prove Theorem 2, we introduce new independent variables $\chi = x - \varphi(t)$ and $\tau = t$ (with Jacobian equal to 1) and show that in these variables, Problem 2 is a characteristic Cauchy problem of standard form, for which an analog of the Kowalewski theorem is valid [9]. The problem considered is a particular case of the problem of the smooth motion of a piston into a gas because at time $t = t_0$, the piston velocity (which, in this case, is zero at the fixed wall O_1O_2) coincides with the gas velocity at a point (the point C in our case) that lies on both the piston and the given sonic characteristic CD. The solution of Problem 2, which, according to Theorem 2, is defined only in a certain neighborhood of the point C, should be considered only in the region DCE (Fig. 1). This flow results from the interaction of the initial rarefaction wave (solution of Problem 1) with the impermeable wall O_1O_2 and is specified by the law of initial motion of the piston $x = x_p(t)$ and the position of the wall O_1O_2, i.e., the initial thickness of the gas layer. The quantity $x_* - x_0$ is a second arbitrary element in Kraiko's problem.

Problem 3. On the wall O_1O_2, we choose a point E with coordinates (t_1, x_0), where $t_1 > t_0$ (see Fig. 1), that lies in the domain of definition of the solution of Problem 2. The point E is the last, third, arbitrary element in Kraiko's problem. At the chosen point E, the speed of sound c_1 in the solution of Problem 2 is uniquely determined: $c(t_1, x_0) = c_1$. Therefore, the gas density ρ_1 is also uniquely determined at this point E: $\rho_1 = c_1^{2/(\gamma-1)}$. Thus, the value $\rho = \rho_1$ is the last arbitrary element that, together with the function $x = x_p(t)$ and the value, $x_* - x_0$, determines uniquely the solution of Kraiko's problem for the case of a rarefaction wave.

In addition, the choice of the point E determines uniquely the analytic functions $x = \varphi^-(t)$ and $x = \varphi^+(t) \equiv x_0 + c_1(t - t_1)$ and, hence, the trajectories of the sonic characteristic EF of the family C^- of the flow corresponding to the region DCE (solutions of Problem 2) and the characteristic EG (straight line) of the family C^+ of the quiescent homogeneous gas with density ρ_1 corresponding to the region O_2EG (Fig. 1). Next, one obtains uniquely the analytic functions

$$c\Big|_{x=\varphi^-(t)} = c_1^-(t), \quad c_1^-(t_1) = c_1, \quad u\Big|_{x=\varphi^-(t)} = u_1^-(t), \quad u_1^-(t_1) = 0; \tag{1.7}$$

$$c\Big|_{x=\varphi^+(t)} = c_1^+(t) \equiv c_1, \qquad u\Big|_{x=\varphi^+(t)} = u_1^+(t) \equiv 0, \tag{1.8}$$

which specify, respectively, the gas parameters on the characteristic EF of the flow corresponding to the region DCE and the state variables of the quiescent homogeneous gas corresponding to the region O_2EG. In this case, $\dot{\varphi}^\pm(t) = u_1^\mp \pm c_1^\pm(t)$ and $\varphi^\pm(t_1) = x_0$.

Thus, Problem 3 reduces to the Goursat problem (1.1), (1.7), (1.8) in the region FEG (see Fig. 1).

Because of the nonlinearity of system (1.1), it is impossible to use the results of [13] for semilinear systems to prove the existence of a solution of Problem 3. For the interaction of two simple waves in the case $\nu = 0$, system (1.1) admits exact linearization [10]. These results also cannot be used because, for $\nu = 0$, the unknown flow in the region FEG is a simple wave since it is adjacent to a homogeneous state of rest. Probably, the existence of a solution of Problem 3 can be proved by the method used in [14] for conical potential flows that depend on x_1/t and x_2/t.

Below, however, to prove the existence of a solution of Problem 3, we reduce this problem to the problem of the decay of a weak discontinuity [15] (see also [16]), for which the corresponding theorems were proved for the class of piecewise-analytic functions. Besides the proof of the existence of a solution, this approach can be used to study non-one-dimensional solutions.

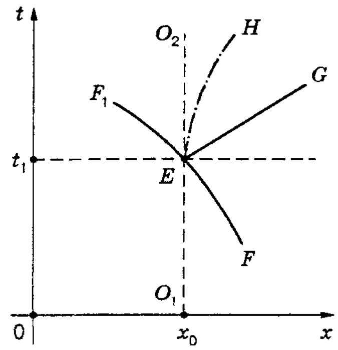

Fig. 2

To match the solution of Problem 2 with other solutions, we can consider it only in a certain part of the domain of definition, e.g., in the domain $DCEF$. However, as noted above, the solution of Problem 2 is defined in a certain neighborhood of the point C. Since the point E is chosen in this domain, there exists a neighborhood of this point $\Omega = \{(t-t_1)^2 + (x-x_0)^2 < \varepsilon^2,\ \varepsilon = \text{const} > 0\}$ in which the solution of Problem 2 is defined by analytic functions. Hence, the characteristic FE can be extended for $x \leqslant x_0$. We consider the characteristic FF_1 (Fig. 2), on which the corresponding gas-dynamic parameters of the solution of Problem 2 are also given, and its part EF_1. It is sufficient to prove the existence of an analytic solution of Problem 3 in a certain neighborhood of the point E in the case where the curves EF_1 and EG (Fig. 2) are taken as the intersecting characteristics that provide the corresponding initial data. The solution of this problem in the region FEG is a constituent of the solution of Kraiko's problem. In this approach, solving Problem 3 is equivalent to solving the following problem of the decay of a weak discontinuity.

We assume that for $t = t_1$ and $x \leqslant x_0$ (see Fig. 2) the gas parameters are

$$U(t,x)\Big|_{t=t_1} = U_{00}^-(x), \tag{1.9}$$

and for $t = t_1$ and $x \geqslant x_0$ they are

$$U(t,x)\Big|_{t=t_1} = U_{00}^+(x); \tag{1.10}$$

they satisfy the continuity condition at the point $x = x_0$:

$$U_{00}^-(x_0) = U_{00}^+(x_0).$$

In Eqs. (1.9) and (1.10), we have $U = \{c, u\}$. The problem is to find the distribution of the gas-dynamic parameters for $t \geqslant t_1$.

Theorem 3. *If functions* (1.9) *and* (1.10) *are analytic in a certain neighborhood of the point* $x = x_0$, *the problem of the decay of a weak discontinuity for* $t \geqslant t_1$ *has a unique piecewise-analytic solution in a certain neighborhood of the point* $(t = t_1,\ x = x_0)$.

Theorem 3 is a particular case of the theorem proved in [15]. In the proof of the above-stated theorem using the initial data (1.9) and (1.10) and Kowalewski's theorem, it is shown that in a certain neighborhood of the point E there exist two background flows given by the analytic functions $U_0^-(t,x)$ and $U_0^+(t,x)$. Let the sonic characteristics C^- and C^+ (curves EF_1 and EG, respectively) be given by the analytic functions $x = \varphi^-(t)$ and $x = \varphi^+(t)$ $[\varphi^-(t_1) = \varphi^+(t_1) = x_0]$. The gas-dynamic parameters of the background flows on these characteristics are also analytic functions and satisfy the continuity condition at the point E:

$$U_0^-(t,x)\Big|_{x=\varphi^-(t)} = U_1^-(t), \quad U_0^+(t,x)\Big|_{x=\varphi^+(t)} = U_1^+(t), \quad U_1^-(t_1) = U_1^+(t_1). \tag{1.11}$$

If, as the background flows, we use the solution of Problem 2 and the solution corresponding to the quiescent homogeneous gas ($\rho = \rho_1$), respectively, then the right sides of Eqs. (1.11) coincide with the right sides of Eqs. (1.7) and (1.8). Therefore, Problem 3 is equivalent to the problem of the decay of a weak discontinuity. To solve the latter, we introduce the unknown curve EH (Fig. 2) which passes through the point E and is given by the unknown function $x = \psi(t)$. The curve EH divides the region F_1EG into the domains F_1EH and HEG. In these domains, the corresponding solutions $U^-(t,x)$ and $U^+(t,x)$ of the system (1.1) are determined. On the characteristics considered, these solutions satisfy the conditions

$$U^-(t,x)\Big|_{x=\varphi^-(t)} = U_1^-(t), \qquad U^+(t,x)\Big|_{x=\varphi^+(t)} = U_1^+(t),$$

i.e., $U^-(t,x)$ must satisfy condition (1.7) and $U^+(t,x)$ must satisfy condition (1.8). In addition, on the curve EH, we require satisfaction of the equality

$$U^-(t,x)\Big|_{x=\psi(t)} = U^+(t,x)\Big|_{x=\psi(t)},$$

which implies that the curve EH is a contact line on which the gas velocities and pressures of the unknown flows $U^-(t,x)$ and $U^+(t,x)$ coincide. For the unknown function $x = \psi(t)$, we formulate the Cauchy problem:

$$\dot\psi(t) = u^+(t,x)\Big|_{x=\psi(t)}, \qquad \psi(t_1) = x_0.$$

Thus, the problem of the decay of a weak discontinuity is formulated as an initial-boundary-value problem. It has been proven [15] that in a certain neighborhood of the point E, a solution of this problem for the five unknown functions $c^-(t,x)$, $u^-(t,x)$, $c^+(t,x)$, $u^+(t,x)$, and $\psi(t)$ exists and is unique. Since the contact line is not a characteristic for one-dimensional isentropic flows, the gas flow in the neighborhood of this line is uniquely determined by specifying gas-dynamic parameters on it. Therefore, if the solutions $U^-(t,x)$ and $U^+(t,x)$ coincide on EH, they coincide in its neighborhood. Thus, in their domains of existence, both flows are given by the same analytic functions $U^-(t,x) \equiv U^+(t,x)$, which are solutions of Problem 3 that are defined in a certain neighborhood of the point E. Therefore, a solution of Kraiko's problem exists in the region FEG also (see Fig. 1).

We choose a certain point on the characteristic EG (see Fig. 1) that lies in the domain of existence of the solution of Kraiko's problem, and from this point we construct the trajectory of the corresponding gas particle (dot-and-dashed curve A_1B_1). Taking into account the first coefficients of the series that give solutions of Problems 1–3, we can prove that the entire curve A_1B_1 lies in the domain of existence of the solution of these problems. This curve is taken as the trajectory of motion of a new impermeable piston. At the points of intersection of the characteristics EF, CD, and CA, the solution has a weak discontinuity (the discontinuity of derivatives not lower than second-order). Hence, it is shown that a solution of Kraiko's problem exists in a certain domain $O_1A_1B_1O_2$ (see Fig. 1). Thus, we have proved that a gas mass can be transferred from one homogeneous state of rest into another by a shock-free method using an impermeable piston.

Discussion of Results. The theorems proved are local in character and, therefore, cannot be used to determine the upper bound for the gas mass for which a solution of Kraiko's problem exists. One can assume that for $\nu = 0$, this bound does not exist. However, to determine the constraint on the mass rigorously (for both $\nu = 0$ and $\nu = 1$ and $\nu = 2$), it is necessary to construct a global flow. For one-dimensional flows, approximate solutions of the equations of gas dynamics can be constructed by the method of characteristics. Using the local theorem proved here, one can pose initial-boundary-value problems and determine the arbitrariness encountered in these problems and the order of solving these problems.

The arbitrariness that arises in the formulation of initial-boundary-value problems allows one to consider various optimization problems (see, e.g., [5, 7]). It is possible that in Kraiko's problem with a rarefaction wave, the specified density ρ_1 can be reached in the fastest time if the solution corresponding to the initial state of rest is matched to a centered rarefaction wave for $\nu = 0$ and to its analogs described in Theorem 1 for $\nu = 1$ and 2.

The proposed approach can be extended to the case where at the initial and final moments, the gas fills cylindrical and spherical regions ($x_0 = 0$). In this case, the condition of no normal flow through the wall $u\Big|_{O_1O_2} = 0$ is replaced by the symmetry condition on the axis ($\nu = 1$) or at the center ($\nu = 2$): $u\Big|_{x=0} = 0$.

In [4–7], flow configurations similar to those considered in the present paper were studied (in Fig. 1, the axis $x = 0$ coincides with the line O_1O_2). However, in the proof of the existence of solutions for the case $x_0 = 0$, difficulties arise at the point C of the characteristic AC which lies on the symmetry axis ($\nu = 1$) or at the center of symmetry ($\nu = 2$). As shown in [17], the first derivatives of the gas-dynamic parameters $(\partial u/\partial x)\big|_{AC}$ and $(\partial \rho/\partial x)\big|_{AC}$ leading out from the line AC go to infinity as $t \to x_* - 0$, i.e., a gradient catastrophe occurs at the point $C(t = x_*, x = 0)$. This singularity at the point C arises when both the rarefaction wave and the compression wave focus on the axis or center of symmetry. At present, for cylindrically and spherically symmetric unsteady flows, there are no proven general statements concerning flow configurations or properties after a weak discontinuity reaches the axis (center) of symmetry. In cases (see [18], [19]) for which there are mathematically rigorous solutions of the problem of the focusing of a compression wave on the axis (center) of symmetry, a shock wave rather than a sonic characteristic is reflected from the point C.

For Problems 1–3, there are extensions to the case of non-one-dimensional flows of a normal gas (see, e.g., [9] and the papers cited therein). For multidimensional flows, Problem 3 is also equivalent to the corresponding problem of the decay of a weak discontinuity. Therefore, the main difficulty in solving Kraiko's problem for the non-one-dimensional case is associated with the construction of a solution of Problem 2 in the class of non-one-dimensional isentropic potential flows subject to the following conditions. On the fixed impermeable wall at $t = t_1$, first, the gas density must be constant and, second, the normal and tangential components of the velocity vector must vanish.

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